Squares and Square Roots

Square Number: Any natural number 'p' which can be represented as y^2, where y is a natural number, then 'p' is called a **Square Number**.

Example

$4 = 2^2$, $9 = 3^2$, $16 = 4^2$

Where 2, 3, 4 are the natural numbers and 4, 9, 16 are the respective square numbers.

Such types of numbers are also known as **Perfect Squares**.

Some of the Square Numbers

1^2	=	1	16^2	=	256
2^2	=	4	17^2	=	289
3^2	=	9	18^2	=	324
4^2	=	16	19^2	=	361
5^2	=	25	20^2	=	400
6^2	=	36	21^2	=	441
7^2	=	49	22^2	=	484
8^2	=	64	23^2	=	529
9^2	=	81	24^2	=	576
10^2	=	100	25^2	=	625
11^2	=	121	26^2	=	676
12^2	=	144	27^2	=	729
13^2	=	169	28^2	=	784
14^2	=	196	29^2	=	841
15^2	=	225	30^2	=	900

Properties of Square Numbers

- We can see that the square numbers are ending with **0, 1, 4, 5, 6 or 9** only**.**
- None of the square number is ending with 2, 3, 7 or 8.
- **Any number having 1 or 9 in its one's place will always have a square ending with 1**.

Number	Square Number
1	1
9	81
11	121
19	361
21	441

- **Any number which has 4 or 6 in its unit's place, its square will always end with 6**.

Number	Square Number
4	16
16	256
24	576
36	1296
44	1936

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- **Any number which has 0 in its unit's place, its square will always have an even number of zeros at the end**.

Number	Square number
10	100
50	2500
100	10000
150	22500
400	160000

Some More Interesting Patterns

1. Numbers between Square Numbers

If we take two consecutive numbers n and n + 1, then there will be (2n) non-perfect square numbers between their square's numbers.

Example

Let's take n = 5 and $5^2 = 25$

n + 1 = 5 + 1 = 6 and $6^2 = 36$

2n = 2(5) = 10

There must be 10 numbers between 25 and 36.

The numbers are 26, 27, 28, 29, 30, 31, 32, 33, 34, 35.

2. Adding Odd Numbers

Sum of first n natural odd numbers is n^2.

$$1 = 1 = 1^2$$
$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$
$$1 + 3 + 5 + 7 + \cdots + n = n^2$$

Any square number must be the sum of consecutive odd numbers starting from 1.

And if any natural number which is not a sum of successive odd natural numbers starting with 1, then it will not be a perfect square.

4. A Sum of Consecutive Natural Numbers

5. The Product of Two Consecutive Even or Odd Natural Numbers

If we have two consecutive odd or even numbers (a + 1) and (a -1) then their product will be $(a^2 - 1)$

Example

Let take two consecutive odd numbers 21 and 23.

Revision Notes $21 \times 23 = (20 - 1) \times (20 + 1) = 20^2 - 1$